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Dj. Kadijevich & R. M. Zbiek
(editors)

Structured Abstracts

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Preface

Computer Algebra in Mathematics Education (CAME) is an open, international organization founded during a special meeting at the 8th International Congress of Mathematical Education (ICME) in Seville in July 1996. Its main goal is to facilitate the dissemination and exchange of information on research and development in the use of computer algebra in mathematics education. Beginning in 1999, CAME biannually organized symposiums. (See www.lkl.ac.uk/research/came/)

The 6th CAME Symposium: Improving tools, tasks and teaching in CAS-based mathematics education, took place in Belgrade, Serbia, on 16 and 17 July 2009, organized by Megatrend University. The three themes of the Symposium, its committees, sponsors and program are listed in the Symposium poster that appears at the end of this booklet. There were sixteen participants from eleven countries and four continents.

This booklet contains the structured abstracts of papers presented at the Symposium. These structured abstracts and the related papers passed a rigorous refereeing process done by members of the Program Committee, whose useful comments and suggestions were intended to help the authors improve these summaries of their papers. We thank these authors for their contributions.

We would like to express our gratitude to Megatrend University for hosting the Symposium and for publishing this booklet of structured abstracts, especially to Prof. Mića Jovanović, Rector of Megatrend University, and Academician Prof. Gradimir Milovanović, dean of Faculty of Computer Science—the host Faculty of the Symposium—as well as to the members of the Organizing Committee. We would also like to express our gratitude to the sponsors, which supported the organization of this Symposium and realized its media presentation.

The Editors

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DESIGNING TASKS FOR CAS CLASSROOMS: CHALLENGES AND OPPORTUNITIES FOR TEACHERS AND RESEARCHERS

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Background. Designing tasks for student use in a CAS classroom presents unique challenges and opportunities for teachers and researchers. In particular, certain aspects of a CAS-based task, not normally considered with a traditional paper and pencil task, may be pivotal in determining the value of the CAS-based task as a resource in mathematical learning.

Aim. The aim of this paper is to isolate those aspects of a CAS-based task which may promote or hinder mathematical activity.

Sources of evidence. Mathematics (including CAS-based mathematical activity) is regarded as a semiotic enterprise. Within this framework, Peirce's notion of mathematical reasoning is used to deconstruct a CAS-based task into three main components: construction of an appropriate sign or diagram, experimentation on this set of signs through manipulations and transformations of signs (written, spoken or imagined), and observation (which of necessity includes interpretation) of the transformed set of signs [1]. A CAS-based task, presented to first-year mathematics university students by the author [2], is deconstructed using this framework. Other examples of CAS-based tasks are also discussed.

Main argument. With regard to the construction of signs, the CAS-based task designer needs to be aware of the type of hybrid knowledge (mathematical and/or syntactical) required by the CAS user to construct appropriate mathematical signs. Where necessary the task design should include guidance on such construction. Or the complexity of the different usages of CAS-based signs can be exploited in the task design. With regard to experimentation, the epistemic value of a CAS-based task may differ widely from the epistemic value of a similar paper and pencil task. Furthermore, the epistemic value of a task may be vastly altered by the provision or withholding of certain information in the task statement. With regard to the interpretation of CAS generated signs, the CAS output may use notation which differs from that of traditional mathemat-

ics notation and so requires special attention by the teacher. Interpretation of graphical representations in a computer environment also entails its own challenges and opportunities.

Conclusion. CAS–based tasks require the construction and interpretation of signs which may have different rules for production and interpretation compared to pencil and paper mathematics though the underlying meaning structure of mathematics is the same in both cases. Accordingly, the designer of a CAS-based task needs to pay particular attention to which signs need to be constructed, how they are to be transformed and how they need to be interpreted in the CAS environment. Finally, complexities around the construction, transformation and interpretation of CAS-based signs may open up new possibilities for task design.

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IMPROVING CAS: CRITICAL AREAS AND ISSUES

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Background. Gjone [1] and Kadijevich [2] addressed several areas and issues that are critical to CAS improvement, particularly regarding relating different representations and problems with autosimplification. However, they do not examine substititions with CAS or CAS compatibility. There is a long and extensive experience in CAS-supported mathematics in Austria (mostly related to TI Derive or other TI-handheld devices) on which the author of this paper draws.

Aim. Having in mind CAS improvement and this Austrian experience, this contribution: (1) reconsiders relating different representations and problems with autosimplification, and (2) examines substitutions with CAS and CAS compatibility.

Sources of evidence. A good, yet not full, linkage of different representations is provided by GeoGebra. Not only maintaining algebraic equivalence but also enabling versatile substitutions is critical to solving some tasks that cannot be (adequately) solved with CAS without using these substitutions. A control of autosimplification, which is missing at present, is not only required by the users who wish to extend built-in CAS features with, for example, user defined functions [2], by also by the teachers who do not want CAS to do some work that should be done by their students. Not only do CAS differ in functions and commands, but also in the effect of copy-and-paste actions from one CAS to the other. Even if displayed as intended, copied material will probably not be understood by the other CAS.

Main argument. Despite some progress, a versatile linking of different representations is not available in present CAS, nor is a control of execution of CAS commands and functions. Substitutions, a special kind of equivalence, are critical to solving problems with CAS. CAS compatibility is not only defendable by learning benefits (e.g., the same command, understood by two CA systems, may give different results promoting conceptual insights), but also by students' need to change CAS used when, for example, the educational institution chooses to use a different CAS, or when the student changes educational institutions.

Conclusion. CAS manufacturers should provide a full linkage of different representations and a control over autosimplification. Substitutions should be made as flexible as possible by using pointing devices, or appropriate commands that recognize the expression structure [3]. Finally, because of its learning benefits and practical constraints, some CAS compatibility should be attained.

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CAS-BASED TASK FRAMEWORKS AND LINKING MULTIPLE REPRESENTATIONS

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Background. The establishment of theoretical frameworks to inform the design of CAS-based tasks provides a recommended structure for the intended role of such tasks in classroom practice. Tool affordances of CAS machines primarily involve symbolic manipulation capabilities, yet connections between symbolic, graphical, and numeric forms are easily achieved with such technology.

Research questions. How can theoretical frameworks be combined to inform the implementation of CAS based tasks? How can the affordances of a CAS be incorporated into task design and implementation to support the flexible use of multiple representations in students' mathematical work?

Sources of Evidence. The task design framework put forth in [1] outlines three main components of a CAS-based task: construction, interpretation, and experimentation. A complementary framework [2] corroborates the French theory (as in [3]) by underlining the co-development of conceptual and technical knowledge through mathematical activity that involves reconciling CAS and paper-and-pencil work, reflecting on these reconciliations, and proving the generalized results. Experience with pre-service secondary mathematics teachers corroborates the need for supporting task design with focused classroom discourse on making flexible use of multiple representations.

Main Argument. The implementation of CAS-based tasks brings about challenges for both teachers and students related to the negotiation of paper-and-pencil and CAS work. In turn, classroom discourse plays a critical role in focusing students' reflection on the reconciliation of the combined by-hand and machine work [3]. When designing a CAS-based task, the salient feature of symbolic manipulation capabilities should be coupled with additional tool affordances, such as graphing, to best foster the flexible use of multiple representations.

Conclusion. The integration of multiple theoretical frameworks for CAS-based task design (e.g., [1, 2]) may lead to a synergistic outcome for versatile classroom implementation of such tasks. The complexities involved in negotiating the use of paper-and-pencil and CAS tools should be given explicit attention in the task design and in classroom discourse. Examining students' ability to flexibly use both graphical and symbolic representations with a CAS-based task that exemplifies the aforementioned theoretical framework is a productive area for future research.

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THE USE OF CAS IN SCHOOL MATHEMATICS: POSSIBILITIES AND LIMITATIONS

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Background. The use of CAS has been an important issue in secondary school mathematics for more than ten years (see www.lkl.ac.uk/research/came/ for that importance). Three topics that are particularly relevant to CAS-based school mathematics are: 1) determining equivalence of algebraic expressions, 2) relating different representations, and 3) making CAS techniques transparent and congruent with their paper-and-pencil versions (see, for example, [1-3] for their relevance). Casio ClassPad provides a versatile CAS environment (www.classpad.org).

Research question. Which features of ClassPad related to the three topics should primarily be improved?

Sources of evidence. By applying the perspective of working mathematically in solving tasks from upper secondary mathematics, we examined the affordances and limitations of commands *judge* and *define*, function *piecewise* and option *GeometryLink*, which are relevant to the three topics, namely: *judge* to determining equivalence of algebraic expressions, *GeometryLink* to relating different representations, and *define* and *stepwise* to making CAS techniques transparent and congruent with their paper-and-pencil versions. Due to theoretical reasons [4], equivalence of expressions cannot be verified in all cases even by ideal CAS.

Main argument. Despite their affordances, all examined features are found limited, particularly command *define*, function *piecewise*, and option *GeometryLink*. In order to do mathematics with CAS in a better way, we need to improve both CAS features (e.g., the three just mentioned) and their use with, for example, user-defined functions.

Conclusion. CAS manufacturers should provide better conditions for the development of user-defined functions by improving CAS functions, commands and options—especially command *define* whereby these functions are defined. Without improving the tool in that way, CAS cannot be used in a functional strategic and pedagogical way as properly required by [5].

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CRITICAL ISSUES OF IMPROVING COMPUTER ALGEBRA SYSTEMS

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Background. Gjone [1] examines the possibilities and limitations of the Casio ClassPad tool for critical topics of determining equivalence of algebraic expressions, relating different representations, and making CAS techniques transparent and congruent with their paper-and-pencil versions. He finds that in order to improve CAS, not only do better built-in features need to be implemented, but also better conditions for creating and using user-defined functions need to be provided. Critical issues related to these requirements are not examined in [1]. Besides Casio ClassPad (www.classpad.org), TI-Nspire CAS (www.ti-nspire.com) is also a powerful CAS environment.

Research question. What are main critical issues regarding the requirements?

Sources of evidence. Solving tasks from upper secondary mathematics in the two environments revealed the following: 1) Editing expressions in the calculator history may not be supported; 2) Managing errors may not be supported; 3) Detailed information regarding managing errors and warnings as well as using commands and functions may not be given in the official documentation; 4) An update command that would update the content of other applications to reflect the changes in the active application is not available; 5) Opportunities for creating user-defined functions may be very limited; 6) Excluding some built-in and user-made affordances is not available.

Main argument. Critical issues in question deal with: 1) updatable lines with commands and functions, 2) controllable execution of commands and functions, 3) richer tool documentation, 4) stronger links between different representations, 5) richer opportunities for creating and using user-defined functions, and 6) customizable user interface.

Conclusion. As the six critical issues identified in this research represent an integrated perspective of recent requirements for CAS improvements [2-4], CAS manufacturers should improve their tools by taking into account these six issues. By using these and other relevant critical issues, further research may study the process of instrumentalization [5] regarding the development of user-made features.

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CAS-BASED TASK REQUIREMENTS AND CRITICAL ACTIVITIES IN COMPLETING THEM

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Background. Critical design features of CAS-based tasks basically deal with constructing signs, using these signs, and interpreting the outcome of their use, and help in solving such tasks may be needed with respect to each of the features [1]. CAS affords regarding these features and their relations and guidance for that help are not detailed in [1].

Research questions. What, in general, may the requirements for CAS-based tasks be? What may critical activities in completing these requirements be?

Sources of evidence. Three basic activities in mathematics are representing, operating and interpreting [2]. Mathematics can thus be viewed as the science of doing and relating these three kinds of mathematization. Critical activities in a transition from one modelling stage to the next describe not only what should happen when a particular transition is achieved with a success, but also what blockages are likely to cause a failure in that transition [3].

Main argument. CAS-based task should require the use of CAS for each of the three kinds of mathematization. Easier tasks should focus on only one mathematization with CAS, gradually adding the other(s) with focus both on them and their relations. By extrapolating from [3], critical activities in completing these tasks may be found in transitions from representing to operating, and from operating to interpreting. The activities—related to mathematics, technology or both—are, for example, using CAS to apply transformations of represented objects, and relating CAS results with mathematical questions aimed to be answered.

Conclusion. Appropriate CAS-based tasks should require representing, operating and interpreting with CAS. Critical activities in completing these tasks, found in the two transitions mentioned above, are related to mathematics, technology or both. As students tend not to coordinate use of mathematics and use of relevant e-tools [4], particular attention should be paid to critical activities related to both mathematics and CAS.

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THE INTEGRATION OF INNOVATIVE CAS SOFTWARE: THEORETICAL FRAMEWORKS AND ISSUES RELATED TO THE TEACHER

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Background. The Casyopée team brings together teachers and researchers to take up the challenge of using Computer Algebra Systems (CAS) to teach about functions at upper secondary level, consistent with recent curricula. The acceptability of CAS by teachers is a central concern. Casyopée is a dynamic geometry and CAS software and one of the six *dynamic digital artefacts* (DDA) developed in the ReMath European project. This project addresses the task of integrating theoretical frames on mathematical learning with digital technologies at the European level, by way of developing and cross experimenting the DDAs. The paper draws on a cross-experiment in Italy and France and on an ongoing study of the dissemination of Casyopée.

Statement of issues. The first issue deals with different ways in which the teacher is taken into account under the influence of theoretical frameworks, when designing classroom activities with Casyopée. The second is about how innovative CAS software can be disseminated beyond the circle of expert teachers.

Sources of evidence. Relative to the first issue, the evidence is taken from the comparison of the scenario designs in the cross-experiment, one influenced mainly by the theory of didactical situations and the instrumental approach, and the other influenced by the semiotic mediation framework. Relative to the second issue, the ongoing study of Casyopée's dissemination distinguishes between three layers of teachers: (1) experts that participated in Casyopée's design, (2) mid-adopters who are interested by using technology in the classroom, but were not associated with Casyopée's development, and (3) all other potential users. The method consists in bringing together six mid-adopters working under the supervision of two experts to create and test classroom situations for potential users. Preliminary findings of the ongoing study of Casyopée's dissemination indicate that mid-adopters need approximately six months to appropriate the software and struggle as they attempt to integrate the tool's constraints. Also, their view of Casyopée contrasts with that of the experts.

Main argument. The comparison between the designs in the cross-experiment contrasts the French and Italian teams. The former designed its experiment through a careful choice of mathematical tasks, overlooking the definition of the teacher's actions. The latter tried to anticipate possible development of the pedagogical plan and to plan some possible canvas for the teachers to manage class discussions. A process of disseminating innovative CAS software beyond the circle of experts is a complex and long term enterprise.

Conclusion. The cross-experiment shows complementary ways of considering the teacher in classroom scenarios: one as a task designer, carefully taking into account the software functionalities for preparing the students' interaction with the software, and the other as a mediator, helping students to draw mathematical meaning from this interaction. This was helpful for writing scenarios for teachers as outcomes of the project. Further work will help people to understand the differences between layers of teachers and how this understanding can be a basis for building communities of teachers and researchers to support dissemination.

SOME ISSUES ON DESIGNING TASKS FOR CAS CLASSROOMS

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Background. There are many problems arising in tasks design in the traditional mathematics education. The use of technology in the teaching process requires a reexamination of these problems as well as brings some new ones.

Research questions. How can an appropriate selection of tools circumvent several shortcomings in the design of CAS tasks? How might we examine the design and evaluation of CAS tasks? How should teachers use CAS tasks?

Sources of evidence. There are many challenges concerning the design of CAS tasks [1]. An appropriate view on learning resources deals with their design, use and modification [2]. Math teachers, especially those in upper primary and secondary schools, want to be in control of the complete didactic situation involving CAS [3].

Main argument. (1) Problems in design of CAS tasks could be seen in a different light if another tool was to be used. If a task designer overcomes his desire "to stay within the same environment", it is possible to design the task in a more flexible way, improving its epistemic value. Also, CAS tasks should promote appropriate CAS use. CAS task should thus be designed in a way that encourages students to choose the most suitable CAS tools. (2) When designing and evaluating CAS tasks, we should envision the whole process of their design, usage and modification. The view of a specific task changes when the didactical situation changes. (3) The authors of materials meant for use by teachers all too often prepare the materials as a monolithic block, constructed in the way an ordinary workbook would be. This demands that the teacher takes them as a whole, precisely in the order they were written in. As the teacher usually serves as an intermediator between the task and the student, he/she should make choices about the materials and combine them into new ways.

Conclusion. CAS tasks should be designed in a flexible way supporting an appropriate use of different CAS. An appropriate view on CAS tasks should deal with the whole process of their design, usage and modification. Teachers should adapt CAS tasks respecting knowledge, skills and needs of their students.

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A WAY TO IMPROVE THE USE OF CAS FOR INTEGRATION

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Background. Results of CAS integration are sometimes not satisfactory [1], and sometimes even do not exist (despite the existence of the solution in the integral tables). In Mupad and Maple, for example, definite integral $I := \text{int}(1/((1+x^2)(1+e^{2x})), x = -1..1);$ cannot be solved. Until recently CAS only gave final result of integration. A new approach for CAS-supported integration includes interactive single-step computations (e.g. package "Analysis" in MuPad [2]; package "Student" in Maple or Java user graphics interfaces called Maplets in Maple [3]).

Aim. By focusing on the issues of transparency of CAS techniques and their congruence with paper-and-pencil versions of these techniques [4], this study examines limitations of a step-by-step approach to CAS-supported integration and a way to reduce them.

Sources of Evidence. Interactive CAS integration enables shorter solutions. Consider the use of package "Student" in Maple. An example of different solutions of the same integration is given below:

$\int (x-x^2)^{-1/2}\,dx$		$\int (x-x^2)^{-1/2}\,dx$	
$= \int \frac{2}{\sqrt{-4u^2+1}}\,du$	$[change, u = x - \frac{1}{2}, u]$	$= \int 2dt$	$[change, x = \sin^2(t)]$
$= 2\int (-4u^2+1)^{-1/2}\,du$	$[const.multiple]$	$= 2t$	$[const.]$
$= 2\int \frac{1}{2}\,du_1$	$[change, u = \frac{1}{2}\sin(u_1), u_1]$	$= 2\arcsin(\sqrt{x})$	$[revert]$
$= u_1$	$[const.]$		
$= \arcsin(2u)$	$[revert]$		
$= \arcsin(2x-1)$	$[revert]$		

The first solution on the left is obtained directly by Maple integration, whereas the second one on the right is obtained manually by the input of substitution in the first step of the integration. By introducing such substitutions, there is a possibility that CAS supported integration gives more solutions. For the second example, take the above mentioned definite integral, which, as underlined, some CAS tools cannot solve. The result is π/4, and it can obtained by using transformation $1/((1+x^2)(1+e^{2x})) = 1/(1+x^2) - 1/((1+x^2)(1+e^{-2x}))$.

Main Argument. The previously mentioned transformation is an instance of general transformation $\frac{1}{f(x)(1+e^{ax})} = \frac{1}{f(x)} - \frac{1}{f(x)(1+e^{-ax})}$, $a \in R$, which results in $I = \int_{-1}^{1} \frac{1}{f(x)(1+e^{ax})} dx \Rightarrow I = \frac{1}{2}\int_{-1}^{1} \frac{1}{f(x)} dx$, where $1/f(x)$ is an even integrable function over [-1, 1]. Using suitable transformations is critical to improving step-by-step CAS integration.

Conclusion. In order to make CAS techniques transparent and congruent with their paper-and-pencil versions, an interactive step-by-step integration should be used. To obtain (more adequate) solutions by using this approach, new transformations at specific steps should be applied. While research should focus on uncovering these transformations, future CAS implementations should include them in their built-in features.

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TEACHERS EXPERIMENTING WITH TOOLS AND TASKS: FROM FEUERBACH TO APOLLONIUS AND BACK

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Background. Lagrange [1] mentions only briefly that students conjecture about solutions while exploring algebraic expressions that model geometrical objects. Famous problems on tangent circles (e.g., Apollonius'problem of constructing a circle tangent to three given circles and Feuerbach's theorem that the nine-point circle related to a given triangle is tangent to the inscribed and circumscribed circles of that triangle) are suitable for combining Computer Algebra (CAS) and Interactive Geometry (IG) tools.

Aim. We attempt to facilitate experimental learning and deduction by combining work with CAS and IG. We wish to learn with the teachers how to develop technology-based curricular materials that enhance both experimentation and proof. We are also interested to identify effective ways for implementing the developed materials in the schools.

Method. The problems were presented to teachers in professional development courses on Analytic Geometry. A didactic sequence designed starts and ends with Feuerbach's work, linked by the old tangency problem of Apollonius. IG was used for introducing the mathematical situation to the teachers as well as for motivating conjectures and attempts to prove them. By algebraic methods with CAS one can solve Apollonius problem. We suggested to the teachers an experimental approach outsourcing the powerful *implicit plotting* facility of the CAS.

Results. CAS-experienced teachers appreciated the potential of implicit plotting of CAS for experimentation, but also argued for a formal algebraic representation of the results. They experienced cognitive difficulties regarding the conversions of representations, which are, according to Duval [2], critical for learning mathematics. Only a few teachers who participated in the courses chose to present the examined problems to their students.

Conclusion. The use of Computer Algebra (CAS) and Interactive Geometry (IG) tools involves (a) flexibility in transforming representations and (b) integration of experimentation and deduction. It is important to analyze with the teachers examples of transforming discursive, numeric, algebraic, geometrical and graphical registers of representations. Such analysis provides content-based professional development that is, according to Mayes [3], a significant key to dissemination. Presentations of the examined problems in the school should be used to demonstrate to the teachers how the tools elicit conceptual understanding from their students.

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TEACHING WITH A CAS-IG TOOL

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Background. Many CAS and IG (Interactive Geometry) tools exist, but classroom use of the tools is not the norm. The ReMath Project [1] developed a CAS-IG integrated tool called Casyopée to explore algebraic functions as models of geometric dependencies and co-variations. I examine the tool, explore questions concerning teachers adapting the tool to the classroom, and discuss the challenges of disseminating tools to a broad teacher audience.

Research questions. How do we move the use of CAS-IG beyond first-adaptors? What are the barriers to broad dissemination and use of such tools?

Sources of Evidence. The pedagogical plan for development and implementation of Casyopée is based on four theoretical frameworks: setting notion and registers of representations [1], instrumental genesis [2], and theory of didactical situations [3]. If functions are to serve as models in the project, then teachers must understand translation between function representations [4]. This requires content-based sustained professional development the includes diagnosing teacher needs, designing and implementing intervention strategies, and evaluation of impact on teacher's content knowledge and teaching techniques [5].

Main Argument. The concept of associated function is a prerequisite for using Casyopée. This requires a deep understanding of the concept of function translation, which some teachers and many students may not possess. In addition, powerful integrated tools like Casyopée often overwhelm teachers and students with the number of operations they can perform, such as solving equations, graphing, simplifying expressions, and in the case of Casyopée integration of algebra with geometry. Combined with the change in expectations for what it means to learn mathematics that accompanies the use of CAS-IG tools, there are significant barriers to broad dissemination and use of such tools.

Conclusion. The French experiment raises some very important issues concerning the broad dissemination of CAS or IG tools. Until school administrators, teachers, parents, and students accept that teaching for understanding is the central tenant of quality mathematics education, the application of CAS and IG in the classroom will be seen as extraneous. Broad use of CAS-IG tools will require sustained content-based Professional Development (PD) and a change in focus to teaching for understanding that makes use of the tools paramount. Developers may also have to consider creating micro-tools that have fewer options and are focused on a narrow set of concepts.

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6th Symposium

of International Group for Computer Algebra in Mathematics Education:

Improving tools, tasks and teaching in CAS-based mathematics education

Megatrend University, Belgrade, Serbia
16-17 July 2009

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Theme 1: Improving tools
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Coordinator and reactor: Djordje Kadijevich

Theme 2: Improving tasks
Presenter: Margot Berger, *University of Witwatersrand, South Africa*
Coordinator and reactor: Matija Lokar

Theme 3: Improving teaching
Presenter: Jean-baptiste Lagrange, *University of Reims, France*
Coordinator and reactor: Robert Mayes

Symposium Programme
Wednesday, 15 July
19:00-20:00 Welcome cocktail
Thursday, 16 July
9:00- 9:15 Registration
9:15- 9:30 Symposium opening
9:30-10:30 Theme 1 plenary
10:30-11:00 Refreshment break
11:00-12:00 Theme 2 plenary
12:00-13:00 Theme 3 plenary
13:00-14:30 Lunch
14:30-15:00 General discussion
15:00-16:00 Separate work of thematic groups
16:00-16:30 Refreshment break
16:30-17:30 Separate work of thematic groups
17:30-18:00 CASIO presentation
19:00-21:00 CAME dinner
Friday, 17 July
9:00-10:30 Reports of thematic groups
10:30-11:00 Refreshment break
11:00-11:45 Final discussion
11:45-12:15 Future CAME activities
12:15-12:30 Symposium closing
12:30-14:00 Lunch

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